A има най-малка стойност за най-голяма стойност на $\sin 4\alpha$ – тя е 1, т.е. $\sin 4\alpha = 1; 4\alpha = \frac{\pi}{2}; \alpha = \frac{\pi}{8}$;

б) $A = \frac{1 + \cos 2\alpha}{\operatorname{cotg}\frac{\alpha}{2} - \operatorname{tg}\frac{\alpha}{2}} = \frac{2\cos^2\alpha}{\frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} - \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}} = \frac{2\cos^2\alpha}{\frac{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}} =$

$\frac{2\cos^2\alpha.\sin\frac{\alpha}{2}.\cos\frac{\alpha}{2}}{\cos\alpha} = \frac{\cos^2\alpha.\sin\alpha}{\cos\alpha} = \cos\alpha.\sin\alpha$; $A = \frac{1}{2}\sin 2\alpha$ – има най-голяма стойност за $\sin 2\alpha = 1 \to 2\alpha = \frac{\pi}{2}; \alpha = \frac{\pi}{4}; A = \frac{1}{2}$.

Стр. 151, Зад. 9.

а) $\sin 5\alpha \sin 4\alpha + \sin 4\alpha \sin 3\alpha - \sin 2\alpha \sin\alpha - 2\sin 5\alpha \sin 3\alpha \cos\alpha =$

$\frac{1}{2}\{\cos(5\alpha - 4\alpha) - \cos(5\alpha + 4\alpha) + \cos(4\alpha - 3\alpha) -$

$(\cos 4\alpha + 3\alpha) - [\cos(2\alpha - \alpha) - \cos(2\alpha + \alpha)]\} -$

$[\cos(5\alpha - 3\alpha) - \cos(5\alpha + 3\alpha)].\cos\alpha =$

$\frac{1}{2}[\cos\alpha - \cos 9\alpha + \cos\alpha - \cos 7\alpha - \cos\alpha + \cos 3\alpha] -$

$\cos 2\alpha.\cos\alpha + \cos 8\alpha.\cos\alpha =$

$\frac{1}{2}(\cos\alpha - \cos 9\alpha - \cos 7\alpha + \cos 3\alpha) - \frac{1}{2}[\cos(2\alpha + \alpha) + \cos(2\alpha - \alpha)] + \cos(8\alpha +$

$\alpha) + \cos(8\alpha - \alpha) =$

$\frac{1}{2}(\cos\alpha - \cos 9\alpha - \cos 7\alpha + \cos 3\alpha - \cos 3\alpha - \cos\alpha - \cos\alpha + \cos 7\alpha) = 0$

б) $\cos\alpha + \cos 3\alpha + \cos 5\alpha + \cos 7\alpha = (\cos\alpha + \cos 7\alpha) + (\cos 3\alpha + \cos 5\alpha) =$

$2\cos\frac{\alpha + 7\alpha}{2}\cos\frac{7\alpha - \alpha}{2} + 2\cos\frac{3\alpha + 5\alpha}{2}\cos\frac{5\alpha - 3\alpha}{2} =$

$2\cos 4\alpha\cos 3\alpha + 2\cos 4\alpha\cos\alpha = 2\cos 4\alpha(\cos 3\alpha + \cos\alpha) =$

$2\cos 4\alpha.2\cos\frac{3\alpha + \alpha}{2}\cos\frac{3\alpha - \alpha}{2} = 2\cos 4\alpha.2.\cos 2\alpha\cos\alpha =$

$2\cos 4\alpha\cos 2\alpha.2.\cos\alpha.\frac{\sin\alpha}{\sin\alpha} = 2\cos 4\alpha\cos 2\alpha.\frac{\sin 2\alpha}{\sin\alpha} =$

$\frac{2.\cos 4\alpha.\sin 4\alpha}{2.\sin\alpha} = \frac{\sin 8\alpha}{2\sin\alpha}$;

в) $\sin^2\alpha + \sin^2 2\alpha + \sin^2 3\alpha + \sin^2 4\alpha =$

$\frac{1 - \cos 2\alpha}{2} + \frac{1 - \cos 4\alpha}{2} + \frac{1 - \cos 6\alpha}{2} + \frac{1 - \cos 8\alpha}{2} =$

$2 - \frac{1}{2}(\cos 2\alpha + \cos 4\alpha + \cos 6\alpha + \cos 8\alpha) =$

$2 - \frac{1}{2}\left(2\cos\frac{2\alpha + 8\alpha}{2}.\cos\frac{8\alpha - 2\alpha}{2} + 2\cos\frac{4\alpha + 6\alpha}{2}\cos\frac{6\alpha - 4\alpha}{2}\right) =$

$2 - \frac{1}{2}(2\cos 5\alpha.\cos 3\alpha + 2\cos 5\alpha\cos\alpha) = 2 - \frac{2}{2}\cos 5\alpha(\cos 3\alpha + \cos\alpha) =$

$2 - \frac{2}{2}\cos 5\alpha.2\cos\frac{3\alpha + \alpha}{2}\cos\frac{3\alpha - \alpha}{2} = 2 - \frac{4}{2}\cos 5\alpha.\cos 2\alpha\cos\alpha =$

$2 - \frac{2}{2}\cos 5\alpha.\cos 2\alpha.2\cos\alpha.\frac{\sin\alpha}{\sin\alpha} = 2 - 2\cos 5\alpha.\cos 2\alpha.\frac{\sin 2\alpha}{2\sin\alpha} =$

$2 - \cos 5\alpha\frac{\sin 4\alpha}{2\sin\alpha}$.

г) $\operatorname{tg}\frac{\alpha + \beta}{2}.\operatorname{tg}\frac{\beta + \gamma}{2}.\operatorname{tg}\frac{\gamma + \alpha}{2} = \frac{\sin\frac{\alpha + \beta}{2}}{\cos\frac{\alpha + \beta}{2}}.\frac{\sin\frac{\beta + \gamma}{2}}{\cos\frac{\beta + \gamma}{2}}.\frac{\sin\frac{\gamma + \alpha}{2}}{\cos\frac{\gamma + \alpha}{2}} =$

$\frac{\sin\frac{\alpha + \beta}{2}\frac{1}{2}\left[\cos\left(\frac{\beta + \gamma}{2} - \frac{\gamma + \alpha}{2}\right) - \cos\left(\frac{\beta + \gamma}{2} + \frac{\gamma + \alpha}{2}\right)\right]}{\cos\frac{\alpha + \beta}{2}\frac{1}{2}\left[\cos\left(\frac{\beta + \gamma}{2} - \frac{\gamma + \alpha}{2}\right) + \cos\left(\frac{\beta + \gamma}{2} + \frac{\gamma + \alpha}{2}\right)\right]} =$

$\frac{\sin\frac{\alpha + \beta}{2}.\cos\frac{\beta - \alpha}{2} - \sin\frac{\alpha + \beta}{2}.\cos\left(\frac{\alpha + \beta + 2\gamma}{2}\right)}{\cos\frac{\alpha + \beta}{2}.\cos\frac{\beta - \alpha}{2} + \cos\frac{\alpha + \beta}{2}.\cos\left(\frac{\alpha + \beta + 2\gamma}{2}\right)} =$

$\frac{\sin\frac{\alpha + \beta}{2}.\cos\frac{\beta - \alpha}{2} - \frac{1}{2}\left[\sin\left(\frac{\alpha + \beta}{2} + \frac{\alpha + \beta + 2\gamma}{2}\right) + \sin\left(\frac{\alpha + \beta}{2} - \frac{\alpha + \beta + 2\gamma}{2}\right)\right]}{\cos\frac{\alpha + \beta}{2}.\cos\frac{\beta - \alpha}{2} + \frac{1}{2}\left[\cos\left(\frac{\alpha + \beta}{2} - \frac{\alpha + \beta + 2\gamma}{2}\right) + \cos\left(\frac{\alpha + \beta}{2} + \frac{\alpha + \beta + 2\gamma}{2}\right)\right]} =$

$\frac{\frac{1}{2}\left[\sin\left(\frac{\alpha + \beta}{2} + \frac{\beta - \alpha}{2}\right) + \sin\left(\frac{\alpha + \beta}{2} - \frac{\beta - \alpha}{2}\right)\right] - \frac{1}{2}[\sin(\alpha + \beta + \gamma) - \sin(-\gamma)]}{\frac{1}{2}\left[\cos\left(\frac{\alpha + \beta}{2} - \frac{\beta - \alpha}{2}\right) + \cos\left(\frac{\alpha + \beta}{2} + \frac{\beta - \alpha}{2}\right)\right] + \frac{1}{2}[\cos(-\gamma) + \cos(\alpha + \beta + \gamma)]} =$

$\frac{\frac{1}{2}[\sin\beta + \sin\alpha - \sin(\alpha + \beta + \gamma) - \sin(-\gamma)]}{\frac{1}{2}[\cos\alpha + \cos\beta + \cos(-\gamma) + \cos(\alpha + \beta + \gamma)]} =$

$\frac{\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha + \beta + \gamma)}{\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha + \beta + \gamma)}$.

Стр. 151, Зад. 10.

(1) $8x^3 - 6x + 1 = 0$; $x_1 = \cos\frac{2\pi}{9}, x_2 = \cos\frac{4\pi}{9}, x_3 = \cos\frac{8\pi}{9}$. Тогава корените, заместени в (1) го удовлетворяват или

$8\left(\cos\frac{2\pi}{9}\right)^3 - 6.\cos\frac{2\pi}{9} + 1 = 0;$

$2\left[4\left(\cos\frac{2\pi}{9}\right)^3 - 3\cos\frac{2\pi}{9}\right] + 1 = 2.\cos\frac{3.2\pi}{9} + 1 = 2.\cos\frac{2\pi}{3} + 1 =$

$2.\left(-\frac{1}{2}\right) + 1 = 0$

Сл. x_1 е корен. Нека x_2 е корен. Тогава

$8.\left(\cos\frac{4\pi}{9}\right)^3 - 6.\cos\frac{4\pi}{9} + 1 = 2\left[4\left(\cos\frac{4\pi}{9}\right)^3 - 3\cos\frac{4\pi}{9}\right] + 1 =$

$2.\cos 3.\frac{4\pi}{9} + 1 = 2\cos\frac{4}{3}\pi + 1 = 2.\cos\left(\pi + \frac{\pi}{3}\right) + 1 =$

$-2\cos\frac{\pi}{3} + 1 = -2.\frac{1}{2} + 1 = 0.$

Сл. x_2 е корен. Нека x_3 е корен. Сл. $8.\left(\cos\frac{8\pi}{9}\right)^3 - 6\cos\frac{8\pi}{9} + 1 =$

$2\left[4\left(\cos\frac{8\pi}{9}\right)^3 - 3\cos\frac{8\pi}{9}\right] + 1 = 2\cos 3.\frac{8\pi}{9} + 1 =$

$2\cos\frac{8\pi}{3} + 1 = 2\cos\left[2\pi + \frac{2\pi}{3}\right] + 1 = 2\cos\frac{2\pi}{3} + 1 = 2.\left(-\frac{1}{2}\right) + 1 = 0.$

Сл. x_3 е корен.

Стр. 151, Зад. 11.

$\sin\frac{\alpha-\beta}{2} + \sin\frac{\alpha-\gamma}{2} + \sin\frac{3\alpha}{2} = \frac{3}{2}.$

От $\alpha+\beta+\gamma = 180°$ и условието на задачата е видно, че $\beta = \gamma$, защото β и γ имат еднакво разположение (симетричност). Заместваме γ с β в условието на задачата:

$\sin\frac{\alpha-\beta}{2} + \sin\frac{\alpha-\beta}{2} + \sin\frac{3\alpha}{2} = \frac{3}{2}$

(1) $2\sin\frac{\alpha-\beta}{2} + \sin\frac{3\alpha}{2} = \frac{3}{2}; 2\beta + \alpha = 180°; \alpha = 180° - 2\beta;$

$2\sin\frac{180° - 2\beta - \beta}{2} + \sin\frac{3(180° - 2\beta)}{2} = \frac{3}{2};$

$2\sin\left(90° - \frac{3\beta}{2}\right) + \sin(270° - 3\beta) = \frac{3}{2};$

$2\cos\frac{3\beta}{2} - \cos 3\beta = \frac{3}{2}; 2\cos\frac{3\beta}{2} - \left(2\cos^2\frac{3\beta}{2} - 1\right) = \frac{3}{2};$

Полагаме $\cos\frac{3\beta}{2} = y; 2y - (2y^2 - 1) = \frac{3}{2}\ 2y - 2y^2 + 1 - \frac{3}{2} = 0;$

$2y^2 - 2y + \frac{1}{2} = 0; 4y^2 - 4y + 1 = 0;$

$y_{1,2} = \frac{2 \pm \sqrt{4-4}}{4} = \frac{1}{2}; \cos\frac{3\beta}{2} = \frac{1}{2}; \frac{3\beta}{2} = 60°; 3\beta = 120°; \beta = 40°;$

$\alpha = 180° - 2.40° = 100°.$

Тестови задачи

Стр. 151, Зад. 1. а)

Стр. 151, Зад. 2. в) Стойностите на косинуси на ъгли са правилни дроби и произведението им е число по абсолютна стойност по-малко от 1. $\cos 20°$ и $\cos 40°$ са положителни числа, а $\cos 140°$ и $\cos 160°$ са отрицателни числа и сл. произведението на всички числа е положително число и $\in (0; 1)$

Стр. 152, Зад. 4 б, 5 в, 6 г. $\alpha \neq \pm 45 + 180k, \alpha \neq k\pi$

7 а) 8 б) $\left(\frac{2}{2}\sin\alpha\cos\alpha\right)\cos 2\alpha\cos 4\alpha\cos 8\alpha =$

$\left(\frac{2}{2}\frac{\sin}{2}2\alpha\cos 2\alpha\right)\cos 4\alpha\cos 8\alpha =$

$\left(\frac{2}{2}.\frac{\sin 4\alpha}{4}\cos 4\alpha\right)\cos 8\alpha = \frac{2}{2}.\frac{\sin 8\alpha\cos 8\alpha}{8} = \frac{\sin 16\alpha}{16}; n = 16.$

9 г) $\text{tg}^2\frac{\pi}{8} = \frac{\sin^2\frac{\pi}{8}}{\cos^2\frac{\pi}{8}} = \frac{1-\cos 45°}{1+45°} = \frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{2+\sqrt{2}} =$

$\frac{(2-\sqrt{2})(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})} = \frac{4-2\sqrt{2}+2}{4-2} = \frac{6-2\sqrt{2}}{2} = 3-2\sqrt{2}.$

$\text{cotg}^2\frac{\pi}{8} = \frac{1}{\text{tg}^2\frac{\pi}{8}} = \frac{1}{3-2\sqrt{2}} = \frac{1}{3-2\sqrt{2}}.\frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{3+2\sqrt{2}}{9-8} = 3+2\sqrt{2}$

а) $x^2 - x + 6 = 0; x_{1,2} = \frac{1\pm\sqrt{1-4.6}}{2}$ – $D < 0$ – няма решение.

б) $x^2 + 6x + 1 = 0; x_{1,2} = -3 \pm \sqrt{9-1} = -3 \pm 2\sqrt{2} \neq 3 \pm 2\sqrt{2}$ – не са корени.

в) $x^2 - x - 6 = 0, x_{1,2} = \frac{1\pm\sqrt{1+24}}{2} = \frac{1\pm 5}{2}$; не са корени

г) $x^2 - 6x + 1 = 0; x_{1,2} = 3 \pm \sqrt{9-1} = 3 \pm 2\sqrt{2}$ – корени са.