$$\left|\begin{array}{l} 28 = (b-3)^2 + a^2 - 2a(b-3).\dfrac{1}{2} \\ 31 = b^2 + a^2 - 2ab.\dfrac{1}{2} \end{array}\right. \to$$

$$\left|\begin{array}{l} 28 = b^2 - 6b + 9 + a^2 - ab + 3a \\ 31 = b^2 + a^2 - ab \end{array}\right. \to a^2 + b^2 = 31 + ab \to$$ заместваме го в I уравнение на системата:

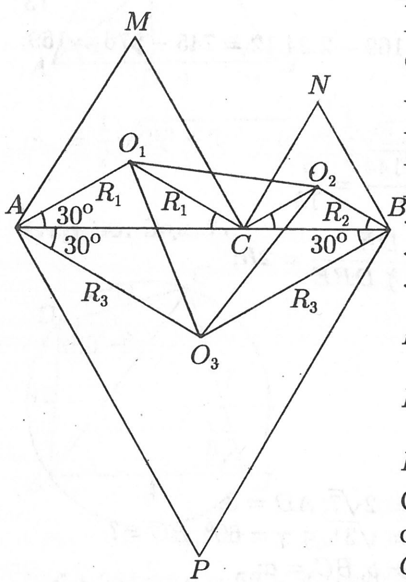
$28 = 31 + ab - 6b + 9 - ab + 3a$; $6b - 3a = 12 \to 2b - a = 4 \to b = \dfrac{4+a}{2}$.

Заместваме в $a^2 + b^2 = 31 + ab$ и получаваме

$a^2 + \left(\dfrac{4+a}{2}\right)^2 = 31 + a\left(\dfrac{4+a}{2}\right)$; $4a^2 + 16 + 8a + a^2 = 124 + 8a + 2a^2$;

$3a^2 = 108$; $a^2 = 36$; $a = 6$.

Стр. 86, Зад. 10.



ACM, BCN, ABP – равностранни $\triangle$ – и, O_1, O_2, O_3 – са центровете им. Означаваме

$AC = a, BC = b$, $AB = a + b$; $R_1 = AO_1 = CO_1$;

$R_2 = O_2C = O_2B$;

$R_3 = AO_3 = BO_3$.

$\sphericalangle O_1CO_2 = 180° - (30° + 30°) = 120°$;

$\sphericalangle O_2BO_3 = 30° + 30° = 60°$;

$\sphericalangle O_1AO_3 = 30° + 30° = 60°$;

$R_1 = \dfrac{a\sqrt{3}}{3}$,

$R_2 = \dfrac{b\sqrt{3}}{3}$,

$R_3 = (a+b)\dfrac{\sqrt{3}}{3}$.

От $\triangle O_1CO_2$, $\triangle CBO_2$ и $\triangle ABO_3$ и косинусовата теорема $O_1O_2^2 = O_1C^2 + O_2C^2 + O_2C^2 - 2O_1C.O_2C\cos 120°$ или

$$O_1O_2^2 = R_1^2 + R_2^2 - 2R_1R_2\left(-\frac{1}{2}\right) = R_1^2 + R_2^2 + R_1R_2;$$

$$O_1O_2^2 = \left(\frac{a\sqrt{3}}{3}\right)^2 + \left(\frac{b\sqrt{3}}{3}\right)^2 + \frac{a\sqrt{3}}{3}.\frac{b\sqrt{3}}{3} = \frac{a^2}{3} + \frac{b^2}{3} + \frac{ab}{3};$$

$O_2O_3^2 = O_2B^2 + O_3B^2 - 2O_2B.O_3B\cos 60° =$

$R_2^2 + R_3^2 - 2R_2R_3.\dfrac{1}{2} = \dfrac{b^2}{3} + \dfrac{a+b)}{3} - \dfrac{b(a+b)}{3} =$

$\dfrac{b^2 + a^2 + 2ab + b^2 - ab - b^2}{3} = \dfrac{a^2 + b^2 + ab}{3}$.

$O_1O_3^2 = AO_1^2 + AO_3^2 - 2AO_1.AO_3.\cos 60° = R_1^2 + R_3^2 - 2R_1R_3.\dfrac{1}{2} =$

$\dfrac{a^2}{3} + \dfrac{(a+b)^2}{3} - \dfrac{a(b+a)}{3} = \dfrac{a^2 + b^2 + a^2 + 2ab - ab - a^2}{3} =$

$\dfrac{a^2 + b^2 + ab}{3}$; $O_1O_2 = O_2O_3 = O_1O_3$ и сл. $O_1O_2O_3$ е равностранен $\triangle$.

Стр. 88, Зад. 1.

а) $c = 5; \alpha = 60, \beta = 45; \gamma = 180° - (60° + 45°) = 75°$;

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$; $\dfrac{a}{\sin 60°} = \dfrac{b}{\sin 45°} = \dfrac{5}{\sin 75°}$;

$a = \dfrac{5\sin 60°}{\sin 75°} = \dfrac{5.\sqrt{3}}{2\sin 75°}$; $b = \dfrac{5\sin 45°}{\sin 75°} = \dfrac{5.\sqrt{2}}{2\sin 75°}$;

б) $c = 7, \alpha = 30°, \beta = 45°, \gamma = 180° - (30° + 45°) = 105°$;

$\dfrac{a}{\sin 30°} = \dfrac{b}{\sin 45°} = \dfrac{7}{\sin 105°}$; $a = \dfrac{7\sin 30°}{\sin 105°} = \dfrac{7.1}{2\sin(180° - 75°)} = \dfrac{7}{2\sin 75°}$;

$b = \dfrac{7\sin 45°}{\sin 105°} = \dfrac{7.\sqrt{2}}{2\sin 107°} = \dfrac{7\sqrt{2}}{2\sin 75°}$;

в) $c = 10, \alpha = 75°, \beta = 45°; \gamma = 180° - (75° + 45°) = 60°$;

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$; $\dfrac{a}{\sin 75°} = \dfrac{b}{\sin 45°} = \dfrac{10}{\sin 60°}$;

$a = \dfrac{10\sin 75°}{\sin 60°} = \dfrac{2.10\sin 75°}{\sqrt{3}} = \dfrac{20}{3}\sin 75°.\sqrt{3}$;

$b = \dfrac{10.\sin 45°}{\sin 60°} = \dfrac{10.\sqrt{2}}{2.\sqrt{3}}.2 = 10\sqrt{\dfrac{2}{3}} = \dfrac{10}{3}.\sqrt{6}$;

г) $c = 3, \alpha = \varphi, \beta = 45°$,

$\gamma = 180° - (\alpha + \beta) = 180° - (\varphi + 45°) = 135° - \varphi$;

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$; $\dfrac{a}{\sin\varphi} = \dfrac{b}{\sin 45°} = \dfrac{3}{\sin(135° - \varphi)}$;

$a = \dfrac{b\sin\varphi}{\sin(135° - \varphi)}$; $b = \dfrac{3\sin 45°}{\sin(135° - \varphi)} = \dfrac{3\sqrt{2}}{2\sin(135° - \varphi)}$;

д) $c = p, \alpha = 60°; \beta = \delta; \gamma = 180° - (60° + \delta) = 120° - \delta$;

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$; $\dfrac{a}{\sin 60°} = \dfrac{b}{\sin\delta} = \dfrac{p}{\sin(120° - \delta)}$;

$a = \dfrac{\sin 60°.p}{\sin(120° - \delta)} = \dfrac{\sqrt{3}p}{2\sin(120° - \delta)}$; $b = \dfrac{p \sin\delta}{\sin(120° - \delta)}$.

Стр. 88, Зад. 2.

а) $a = 5, b = 3, \alpha = 60°$

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$; $\dfrac{5}{\sin 60°} = \dfrac{3}{\sin\beta}$;

$\sin\beta = \dfrac{3\sin 60°}{5} = \dfrac{3.\sqrt{3}}{2.5} = \dfrac{3}{10}\sqrt{3} = \dfrac{3}{10}.0,8660 = 0,2598; \beta = 15°3'$

$\gamma = 180° - (60° + 15°3') = 180° - 75°3' = 104°57'$

$\dfrac{c}{\sin\gamma} = \dfrac{a}{\sin\alpha}$; $c = \dfrac{a\sin\gamma}{\sin\alpha} = \dfrac{5.\sin 104°57'}{\sin 60°} = \dfrac{5.\sin 75°3'}{08660} = \dfrac{5.0,9662}{0,8660} =$ 5,58;

б) $a = 7, b = 12, \alpha = 30°$;

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$;

$\sin\beta = \dfrac{b\sin\alpha}{a} = \dfrac{12\sin 30°}{7} = \dfrac{12}{7.2} = \dfrac{6}{7} = 0,8571$

$B = 58°59'$; $\gamma = 180° - (30° + 58°59') = 180° - 88°59' = 91°1'$;

$\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$;

$c = \dfrac{a\sin\gamma}{\sin\alpha} = 7\sin 91°1'\sin 30° = \dfrac{7.\sin 88°59'}{\frac{1}{2}} = 14.0,9998 \approx 14$

в) $a = 8, b = 10, \alpha = 45°$; $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$; $\sin\beta = \dfrac{b\sin\alpha}{a} = \dfrac{10}{8}\sin 45° =$ $\dfrac{10}{8}.0,7071 = 0,8839$;

$\beta = 62°7'$, $\gamma = 180° - (45° + 62°7') = 180° - 107°7' = 72°53'$;

$\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$; $c = \dfrac{a\sin\gamma}{\sin\alpha} = \dfrac{8\sin 72°53'}{\sin 45°} = \dfrac{8.0,9557}{0,7071} = 10,81$;

г) $a =.5, b = 6, \alpha = \varphi$;

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$; $\sin\beta = \dfrac{b\sin\alpha}{a} = \dfrac{6}{5}\sin\varphi$;

$\gamma = 180° - (\alpha + \beta)$; $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$; $c = \dfrac{a\sin\gamma}{\sin\alpha}$

д) $a = p, b = 8, \alpha = 120°$;

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$; $\sin\beta = \dfrac{b\sin\alpha}{a} = \dfrac{8}{p}\sin 120° = \dfrac{8\sqrt{3}}{p.2}$

$\gamma = 180° - (\alpha + \beta)$; $c = \dfrac{a\sin\gamma}{\sin\alpha}$;

е) и ж) се решават по същия начин.

Стр. 88, Зад. 3.

а) $a = 21; b = 13, \gamma = 30°; c =?, \alpha, \beta =?$

От косинусова теорема $c^2 = a^2 + b^2 - 2ab\cos\gamma$;

$c^2 = 21^2 + 13^2 - 2.21.13\cos 30° = 441 + 169 - 42.13.\dfrac{\sqrt{3}}{2} =$

$610 - \dfrac{546}{2}.1,7320 = 610 - 273.1,7320 =$

$610 - 472,84 = 137,16; c = \sqrt{137,16} = 11,71$;

От синусова теорема $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$; $\dfrac{21}{\sin\alpha} = \dfrac{11,71}{\sin 30°}$;

$\sin\alpha = \dfrac{21.\frac{1}{2}}{11,71} = \dfrac{10,5}{11,71} = 0,8967; \alpha = 63°43'$

$\beta = 180° - (\alpha - \gamma) = 180° - (30° + 63°43') = 180° - 93°43' = 86°17'$

б) $a = 5, b = 8, \gamma = 60°$;

$c^2 = a^2 + b^2 - 2ab\cos\gamma = 5^2 + 8^2 - 2.5.8.\cos 60° = 25 + 64 - 80.\dfrac{1}{2} = 49$;

$c = \sqrt{49} = 7$;

$\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$; $\sin\alpha = \dfrac{a\sin\gamma}{c} = \dfrac{5.\frac{1}{2}}{7} = \dfrac{5}{14} = 0,3571$;

$\alpha = 20°56'; \beta = 180° - (\alpha + \gamma) = 180° - (20°565' + 60°) = 180° - 80°56' =$ $99°44'$

в) $a = 11, b = 13, \gamma = 45°$; $c^2 = 11^2 + 132^2 - 2.11.13.\cos 45° = 121 + 169 -$ $286.0,7071 = 290 - 202,23 = 87,77$; $c = 9,37$

$\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$; $\sin\alpha = \dfrac{a\sin\gamma}{c} = \dfrac{11.\sin 45°}{9,37} = \dfrac{11}{9,37}.0,7071 = 0,8301$

$\alpha = 56°6'; \beta = 180° - (\alpha + \gamma) = 180° - (56°6' + 45°) = 180° - 101°6' =$ $98°54'$

г) $a = 5, b = 3, \gamma = 135°$;

От косинусова теорема $c^2 = 5^2 + 3^2 - 2.5.3\cos 135° = 25 + 9 - 30.(-0,7071) =$ $34 + 21,21 = 55,21$; $c = \sqrt{55,21} = 7,43$;

$\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$; $\sin\alpha = \dfrac{a\sin\gamma}{c} = \dfrac{5.0,7071}{7,43} = 0,6825$;

$\alpha = 43°2'$; $\beta = 180° - (\alpha + \gamma) = 180° - (135° + 43°2') = 180° - 178°2' =$ $1°58'$.

д)

$c^2 = 12^2 + 8^2 - 2.12.\cos 120 = 144 + 64 - 24.8.\left(-\dfrac{1}{2}\right) =$