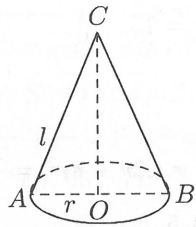


Стр. 65, Зад. 8.



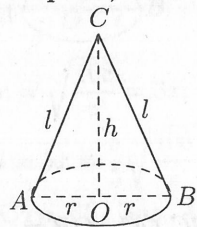
$\triangle ABC$ равностранен; $S_{ABC} = Q.V = ?; 2r = l; h = \frac{a\sqrt{3}}{2}$ - височина в равностранния $\frac{2}{3}$ триъгълник; $OC = h = 2r \frac{\sqrt{3}}{2} = r\sqrt{3} = r\sqrt{3}; \frac{AB \cdot h}{2} = Q$ или $2r \frac{r\sqrt{3}}{2} = Q; r^2\sqrt{3} = Q; r = \sqrt{\frac{Q}{\sqrt{3}}}$;

$$V = \frac{\pi r^2 h}{3} = \pi r^2 \cdot \frac{r\sqrt{3}}{3} = \frac{\pi r^3 \sqrt{3}}{3} = \frac{\pi}{3} \cdot \sqrt{3} \sqrt{\frac{Q^3}{(\sqrt{3})^3}} = \frac{\pi}{3} \frac{Q\sqrt{Q}}{\sqrt{3}} = \frac{\pi Q\sqrt{Q}}{3\sqrt{3}}$$

Стр. 65, Зад. 9.

Виж чертежа на предходната задача. $\sphericalangle ACB = 90^\circ$. Тогава $S_{BC} = \frac{AC \cdot BC}{2} = \frac{l^2}{2}; \frac{l^2}{2} = 9; l^2 = 18; l = 3\sqrt{2}; CO = AO$, защото $\triangle AOC$ - правоъгълен, равностранен триъгълник и следователно $r^2 + h^2 = l^2$ или $2r^2 = l^2; r = \frac{l}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}} = 3; r = h = 3; V = \frac{\pi r^2 h}{3} = \frac{\pi r^3}{3} = \frac{\pi \cdot 3^3}{3} = 9\pi$.

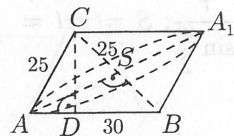
Стр. 65, Зад. 10.



$$\begin{cases} 2l + 2r = 30 \\ \pi r l + \pi r^2 = 60\pi \end{cases} \rightarrow$$

$$\begin{cases} r + l = 15 \\ r l + r^2 = 60 \end{cases}; r(l + r) = 60; r \cdot 15 = 60; r = 4; l = 15 - r = 15 - 4 = 11; AB = 2r = 2 \cdot 4 = 8; AC = BC = l = 11.$$

Стр. 65, Зад. 11.



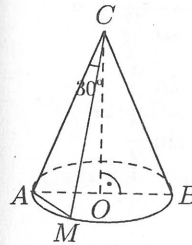
$S_{ABA_1C} = ? AD = 30 : 2 = 15$. От правоъгълен $\triangle ADC$ $CD^2 = AC^2 - AD^2 = 25^2 - 15^2 = 625 - 225 = 400; CD = 20; S_{ABC} = \frac{30 \cdot 20}{2} = \frac{25 \cdot AS}{2}; AS = \frac{30 \cdot 20}{25} = 24;$

$$V = \pi \cdot AS^2 \cdot \frac{CS}{3} + \pi \cdot AS^2 \cdot \frac{SB}{3} = \frac{\pi \cdot AS^2}{3} (CS + SB) = \frac{\pi \cdot AS^2}{3} \cdot BC =$$

$$\pi \cdot \frac{24^2}{3} \cdot 25 = \pi \cdot 8 \cdot 24 \cdot 25 = 4800\pi;$$

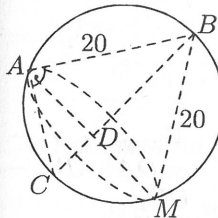
$$S = \pi \cdot AS \cdot AC + \pi \cdot AS \cdot AB = \pi \cdot AS (AC + AB) = \pi \cdot 24 (25 + 30) = \pi \cdot 24 \cdot 55 = 1320\pi.$$

Стр. 65, Зад. 12.



$\triangle ABC$ - равностранен; $A = r; AB = 2r; OC = \frac{s\sqrt{3}}{2}$ - (h в равностранния триъгълник); $OC = \frac{(2r)\sqrt{3}}{2} = r\sqrt{3}; AC = AB = 2r; S_{AMC} = \frac{AC \cdot MC \sin 30^\circ}{2} = \frac{2r \cdot 2r \cdot \frac{1}{2}}{2} = r^2$.

Стр. 65, Зад. 13.



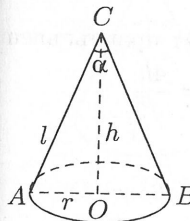
$AB = 20$ - хорда; $BC = 2r = 25; S_{1AMB} = \pi \cdot 20 \cdot DM + \pi CM^2;$ $\sphericalangle CAB = 90^\circ; AC^2 = BC^2 - AB^2 = 25^2 - 20^2 = 225; AC = \sqrt{225} = 15$. От формулата $ab = ch_c$ за правоъгълен триъгълник $AC \cdot AB = BC \cdot AD$ или $15 \cdot 20 = 25 \cdot AD; AD = \frac{15 \cdot 20}{25} = 12;$ $S_{1AMB} = \pi \cdot 20 \cdot 12 + \pi \cdot 12^2 = 384\pi$.

Стр. 65, Зад. 14.

Виж чертежа на предходната задача. $\frac{\pi r^2}{2rh} = \pi; \frac{\pi r^2}{rh} = \pi; \frac{r}{h} = 1; r = h$.

От правоъгълния $\triangle AOC \rightarrow OC : AO = \tan \alpha = \frac{h}{h} = 1; \alpha = 45^\circ$.

Стр. 65, Зад. 15.



$a = r + l \sphericalangle; S_1 = ?; \sphericalangle ACB = \alpha$. От правоъгълния $\triangle AOC, AO : AC = \sin \alpha; AO = l \sin \frac{\alpha}{2}; l + l \sin \frac{\alpha}{2} = a;$ $l \left(1 + \sin \frac{\alpha}{2} \right) = a;$

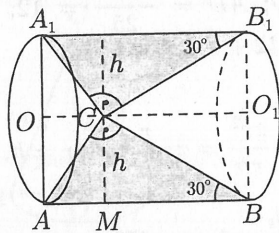
$$l = \frac{a}{1 + \sin \frac{\alpha}{2}}; r = AO = l \sin \frac{\alpha}{2} = \frac{a \sin \frac{\alpha}{2}}{1 + \sin \frac{\alpha}{2}};$$

$$S_1 = \pi r l + \pi r^2 = \pi r(l + r) = \frac{\pi a \sin \frac{\alpha}{2}}{1 + \sin \frac{\alpha}{2}} \left(\frac{a}{1 + \sin \frac{\alpha}{2}} + \frac{a \sin \frac{\alpha}{2}}{1 + \sin \frac{\alpha}{2}} \right) =$$

$$\frac{\pi a \sin \frac{\alpha}{2}}{\left(1 + \sin \frac{\alpha}{2}\right)^2} \left(a + \sin \frac{\alpha}{2}\right) = \frac{\pi a^2 \sin \frac{\alpha}{2} \left(1 + \sin \frac{\alpha}{2}\right)}{\left(1 + \sin \frac{\alpha}{2}\right)^2} = \frac{\pi a^2 \sin \frac{\alpha}{2}}{1 + \sin \frac{\alpha}{2}} =$$

$$\frac{\pi a^2 \sin \frac{\alpha}{2}}{2 \cos^2 \left(45^\circ - \frac{\alpha}{4}\right)}.$$

Стр. 65, Зад. 16.



$$V = V_{\text{цил.}} - V_{\text{конус}AA_1C} - V_{\text{конус}BB_1C} = \pi \cdot OA^2 \cdot AB - \pi \cdot OA^2 \cdot \frac{OC}{3} - \pi \cdot O_1B^2 \cdot \frac{CO_1}{3} =$$

$$\pi h^2 \cdot AB - \left(\frac{\pi h^2}{3} OC + \frac{\pi H^2}{3} \cdot CO_1 \right) =$$

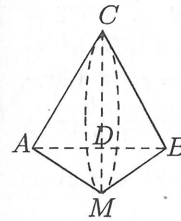
$$\pi h^2 \cdot AB - \frac{\pi h^2}{3} (OC + CO_1) =$$

$$\pi h^2 \cdot AB - \frac{\pi h^2}{3} \cdot OO_1 = \pi h^2 AB - \frac{\pi h^2}{3} AB = \frac{2}{3} \pi H^2 AB.$$

От правоъгълен $\triangle MCB$ $BC = 2MC = 2h - (\sphericalangle 30^\circ)$. От правоъгълен $\triangle ABC$ $BC : AB = \cos 30^\circ$; $2h : AB = \frac{\sqrt{3}}{2}$; $AB = \frac{2h}{\sqrt{3}} \cdot 2 = \frac{4h}{\sqrt{3}}$;

$$V = \frac{2}{3} \pi h^2 AB = \frac{2}{3} \pi h^2 \cdot \frac{4h}{\sqrt{3}} = \frac{8\pi h^3}{3\sqrt{3}} = \frac{8\sqrt{3}}{9} \pi h^3.$$

Стр. 65, Зад. 17.



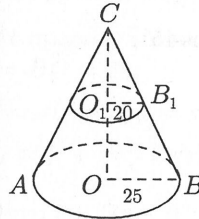
$$\frac{c \cdot CD}{2} = S; CD = \frac{2S}{c}; V = \frac{\pi c}{3} \left(\frac{2S}{c}\right)^2 = \frac{\pi c}{3} \cdot \frac{4S^2}{c^2} = \frac{4\pi S^2}{3} \cdot \frac{1}{c}.$$

$$S_{ABC} = S; AB = c; V = \frac{4}{3} \pi S^2 \cdot \frac{1}{c}?$$

$$V = V_{ACM} + V_{MCB} = \frac{\pi \cdot CD^2 \cdot AD}{3} + \frac{\pi \cdot CD^2 \cdot DB}{3} = \frac{\pi CD^2}{3} (AD + BD) =$$

$$\frac{\pi CD^2}{3} c; \frac{AB \cdot CD}{2} = S;$$

Стр. 69, Зад. 1.



$$h = 20, r = 25; \triangle O_1B_1C \sim \triangle OBC;$$

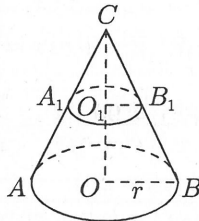
$$O_1B_1 = r_1; CO_1 = x; O_1B_1 : OB = CO_1 : CO;$$

$$r_1 : 25 = x : 20; OO_1 = 12; CO_1 = 20 - 12 = 8 = x;$$

$$r_1 : 25 = 8 : 20; r_1 = \frac{25 \cdot 8}{20} = 10;$$

$$S = \pi r_1^2 = \pi \cdot 100 = 100\pi$$

Стр. 69, Зад. 2.

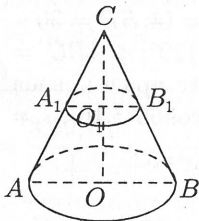


$$\triangle O_1B_1C \sim \triangle OBC \text{ (от } A_1B_1 \parallel AB);$$

$$\frac{O_1B_1}{OB} = \frac{CO_1}{CO} = \frac{1}{2}; O_1B_1 = x;$$

$$\frac{x}{r} = \frac{1}{2}; x = \frac{r}{2}; S = \pi x^2 = \frac{\pi r^2}{4}.$$

Стр. 69, Зад. 3.



$$CO = 12; \frac{\pi x^2}{\pi r^2} = \frac{1}{2} = \frac{h^2}{H^2} = \frac{h^2}{12^2}$$

$$\frac{x^2}{r^2} = \frac{1}{2}; x^2 = \frac{r^2}{2}; x = \frac{r}{\sqrt{2}}; \frac{h^2}{12^2} =$$

$$\frac{1}{2}; h^2 = \frac{144}{2} = 72; h = 6\sqrt{2};$$

$$OO_1 = 12 - 6\sqrt{2} = 12 - 6 \cdot 1,41 = 12 - 8,46 = 3,54.$$