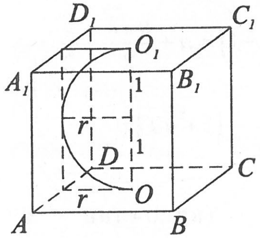
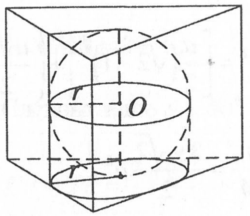


Стр.121, Зад.21.

$r=1;\ h=2.1=2;\ \frac{a}{2}=r=1;\ a=2;$

$S_1=4aH+2a^2=4.2.2+2.2^2=24.$

Стр.121, Зад.22. $V=8\sqrt{3}=BH=\frac{a^2\sqrt{3}}{4}.H;\ r=\frac{H}{2};\ r_{осн.}=\frac{a\sqrt{3}}{6}=R_{кълбо};$

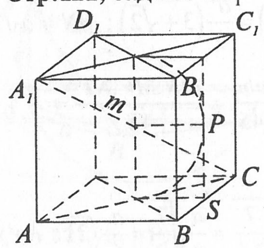


$H=2r=\frac{2a\sqrt{3}}{6};\ V=8\sqrt{3}=\frac{a^2\sqrt{3}}{4}.\frac{2a\sqrt{3}}{6}=\frac{a^3}{4};$

$\frac{a^3}{4}=8\sqrt{3};\ a^3=32\sqrt{3};\ a^3=\frac{8\sqrt{3}.6.4}{2.3}=32\sqrt{3};$

$a=\frac{1}{3}\sqrt[6]{4^2.3.27}=\frac{1}{3}\sqrt[6]{2^4.3^4}=\frac{1}{3}\sqrt[3]{2^2.3^2}=\frac{1}{3}\sqrt[3]{36}=\sqrt[3]{\frac{4}{3}}.$

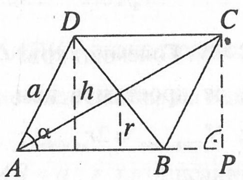
Стр.121, Зад.23. $CA_1=m;\ AC=d;$ От $\Delta ACA_1\ \frac{d}{m}=\cos 30^0=\frac{\sqrt{3}}{2};$



$d=\frac{m\sqrt{3}}{2};\ BC=AB=a;\ H=2r$ или

$H_{призама}=h_{ромб}$ защото сферата е вписана;

$H=\frac{m}{2}$ срещу $\measuredangle 30^0$ в ΔAA_1C; $2r=\frac{m}{2};\ r=\frac{m}{4};$

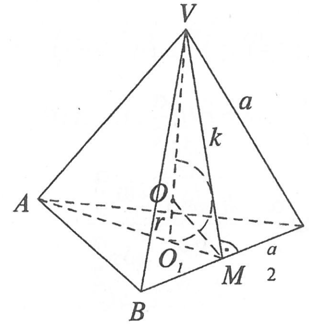


От правоъгълния ΔAPC следва, че $\frac{2r}{d}=\sin\frac{\alpha}{2};$

$\frac{m^2}{2.m\sqrt{3}}=\sin\frac{\alpha}{2};\ \sin\frac{\alpha}{2}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3};$

$\cos\alpha=1-2\sin^2\frac{\alpha}{2}=1-2.\frac{3}{9}=1-\frac{2}{3}=\frac{1}{3}.$

Стр.121, Зад.24. Не.



Стр.121, Зад.25. $O_1M=r_1=\frac{a\sqrt{3}}{6};$

От $\Delta MCV\ \ k^2=a^2-\frac{a^2}{4}=\frac{3a^2}{4};\ k=\frac{a}{2}\sqrt{3};$

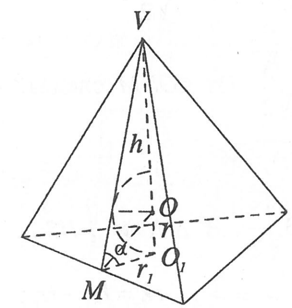
От правоъгълния ΔAO_1V, следва че

$H^2=AV^2-AO_1^2=a^2-\left(\frac{a\sqrt{3}}{3}\right)^2=a^2-\frac{a^2}{3}=\frac{2}{3}a^2$

$H=a\sqrt{\frac{2}{3}};\quad V=\frac{S_n k}{3};\quad r=\frac{3v}{S_n};\quad V=\frac{a^2\sqrt{3}}{4}.\frac{H}{3}=\frac{a^2\sqrt{3}}{4}.a\sqrt{\frac{2}{3}}.\frac{1}{3}=\frac{a^3}{12}\sqrt{2};$

$S_n=\frac{3a.k}{2}+\frac{a^2\sqrt{3}}{4}=3a.\frac{a}{2}\sqrt{3}.\frac{1}{2}+\frac{a^2\sqrt{3}}{4}=\frac{4a^2\sqrt{3}}{4}=\frac{4a^2\sqrt{3}}{4}=a^2\sqrt{3};$

$r=\frac{3V}{S_n}=\frac{3a^3\sqrt{2}}{12a^2\sqrt{3}}=\frac{a\sqrt{6}}{12}.$



Стр.121, Зад.26. а) Дадено h и a; $r=?$

$MO_1=\frac{a\sqrt{3}}{6}$; В $\Delta MO_1V\ \ MO_1$ - ъглополовяща и следователно $O_1O:OV=MO_1:MV$ или

1) $\frac{r}{h-r}=\frac{a\sqrt{3}}{6.MV}$; От правоъгълния ΔMO_1V

$MV^2=MO_1^2+O_1V^2=\left(\frac{a\sqrt{3}}{6}\right)^2+h^2=\frac{a^2}{12}+h^2;$

$\frac{r}{h-r}=\frac{a\sqrt{3}}{6x};\ MV=\sqrt{\frac{a^2}{12}+h^2}\to 1);\ r=\frac{a\sqrt{3}}{6x};\ r.6x=ha\sqrt{3}-a\sqrt{3}r;$

$r\left(6x+a\sqrt{3}\right)=ah\sqrt{3};\ r=\frac{ah\sqrt{3}}{6x+a\sqrt{3}}=\frac{a\sqrt{3}h}{6\sqrt{\frac{a^2}{12}+h^2}+a\sqrt{3}}=$

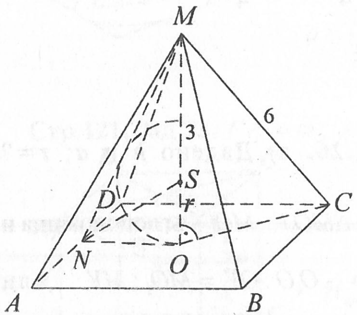
$$= \frac{ah\sqrt{3.12}}{6\sqrt{a^2+12h^2}+6a} = \frac{ah}{\sqrt{a^2+12h^2}+a};$$

б) Дадено h и α, $r = ?$; От ΔMO_1O $r : r_1 = \text{tg}\frac{\alpha}{2}$; 1) $r = r_1\,\text{tg}\frac{\alpha}{2}$; От ΔMO_1V

$$r : h = \text{cotg}\,\alpha;\ r_1 = h\,\text{cotg}\,\alpha \to 1) \to r = h\,\text{cotg}\,\alpha.\text{tg}\frac{\alpha}{2} = \frac{h\cos\alpha}{\sin\alpha}.\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} =$$

$$= \frac{h\cos\alpha\sin\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos^2\frac{\alpha}{2}} = \frac{h\cos\alpha}{2\cos^2\frac{\alpha}{2}}.$$

Стр.121, Зад.27. $\frac{OM}{MC} = \frac{3}{6} = \sin\alpha = \frac{1}{2}$; $\alpha = 30^0$; $\frac{OC}{MC} = \cos 30^0$;



$$OC = MC\cos 30^0 = \frac{6\sqrt{3}}{2} = 3\sqrt{3};$$

$$OC = 3\sqrt{3} = \frac{a\sqrt{2}}{2} \to a = 6\sqrt{\frac{3}{2}};$$

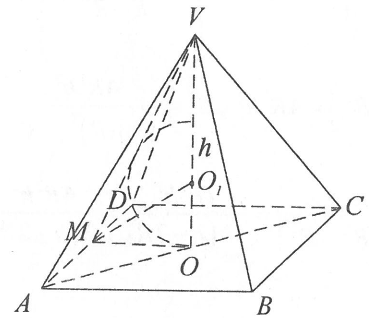
$NO = r = \frac{a}{2} = 3\sqrt{\frac{3}{2}}$. От ΔONM следва, че $MN^2 = NO^2 + OM^2$;

$MN^2 = \left(3\sqrt{\frac{3}{2}}\right)^2 + 3^2 = \frac{27}{2} + 9 = \frac{45}{2}$; $MN = \sqrt{\frac{45}{2}} = 3\sqrt{\frac{5}{2}}$. В ΔNOM, NS - ъглополовяща $\Rightarrow$ $OS : SM = NO : NM$; $\frac{r}{h-r} = \frac{r_1}{k}$; $rk = hr_1 - rr_1$;

$$r(k + r_1) = hr_1;\ r = \frac{hr_1}{k+r_1} = \frac{3.3\sqrt{\frac{3}{2}}}{3\sqrt{\frac{5}{2}} + 3\sqrt{\frac{3}{2}}} = \frac{9\sqrt{6}}{2\left(\frac{3}{2}\sqrt{10} + \frac{3}{2}\sqrt{6}\right)} = \frac{9\sqrt{6}}{3\sqrt{10}+3\sqrt{6}} =$$

$$= \frac{3\sqrt{6}}{\sqrt{10}+\sqrt{6}}.\frac{\sqrt{10}-\sqrt{6}}{\sqrt{10}-\sqrt{6}} = \frac{3\sqrt{6}\left(\sqrt{10}-\sqrt{6}\right)}{10-6} = \frac{3}{4}\sqrt{6}\left(\sqrt{10}-\sqrt{6}\right) =$$

$$= \frac{3}{4}\left(\sqrt{60}-6\right) = \frac{3}{4}\left(2\sqrt{15}-6\right) = \frac{3}{2}\left(\sqrt{15}-3\right).$$



Стр.121, Зад.28. $\measuredangle OMV = \alpha$

От правоъгълен ΔMOV следва, че $MO : OV = \text{cotg}\,\alpha$; $MO = h\,\text{cotg}\,\alpha$;

От ΔMOO_1 следва, че $OO_1 : MO = \text{tg}\frac{\alpha}{2}$;

$$r = OO_1 = MO\,\text{tg}\frac{\alpha}{2} = h\,\text{cotg}\,\alpha.\text{tg}\frac{\alpha}{2}.$$

Стр.121, Зад.29. Виж горния чертеж. ΔABV - равностранен и $MV = \frac{a\sqrt{3}}{2}$ (h в равностранния триъгълник); $MO = \frac{a}{2}$; $\frac{MO}{MV} = \cos\alpha$;

$\cos\alpha = \frac{a.2}{2.a\sqrt{3}} = \frac{\sqrt{3}}{3}$. От правоъгълния ΔOO_1M $OO_1 : OM = \text{tg}\frac{\alpha}{2}$;

$O_1O = OM\,\text{tg}\frac{\alpha}{2} = \frac{a}{2}.\text{tg}\frac{\alpha}{2}$; От формулата $\text{tg}\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$ следва, че

$$\text{tg}\frac{\alpha}{2} = \sqrt{\frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}} = \sqrt{\frac{3-\sqrt{3}}{3+\sqrt{3}}};\ r = OO_1 = \frac{a}{2}\sqrt{\frac{3-\sqrt{3}}{3+\sqrt{3}}} = \frac{a}{2.6}\sqrt{\left(3-\sqrt{3}\right)^2} =$$

$$\frac{a\left(3-\sqrt{3}\right)}{2\sqrt{6}} = \frac{a\left(\sqrt{3}\right)^2}{2\sqrt{2.3}} - \frac{a\sqrt{3}}{2\sqrt{2.3}} = \frac{a}{2}\left(\sqrt{\frac{3}{2}} - \sqrt{\frac{1}{2}}\right) = \frac{a}{4}\left(\sqrt{6}-\sqrt{2}\right).$$

Стр.121, Зад.30. Виж чертежа на задача 28. Дадено h и $R_{сфера}$; $V_{пир.} = ?$

В ΔMOV MO_1 - ъглополовяща и следователно $\frac{MO}{MV} = \frac{OO_1}{O_1V}$ или

$$\frac{a}{2k} = \frac{R}{h-R} \to ah - aR = 2Rk;\ a(h-R) = 2Rk;\ a = \frac{2Rk}{h-R};$$