

$$R = \frac{a\sqrt{2}}{2}; R_1 = \frac{5\sqrt{2}}{2}; R_2 = \frac{3\sqrt{2}}{2}.$$

От правоъг. ΔMCC_1 , където $MC = R - R_1$, $CM = h$, $CC_1 = \ell$,

$$CC_1^2 = MC^2 + h^2; \ell^2 = h^2 + (R_1 - R_2)^2 = 4^2 + \left(\frac{5\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}\right)^2 = 16 + 2 = 18; \ell = \sqrt{18} = 3\sqrt{2}.$$

Стр.50, Зад.22.

$$OC = R = \frac{a\sqrt{3}}{3} = \frac{9\sqrt{3}}{3} = 3\sqrt{3};$$

$$R_1 = O_1C_1 = \frac{a\sqrt{3}}{3} = \frac{3\sqrt{3}}{3} = \sqrt{3};$$

$$SC = R - R_1 = 3\sqrt{3} - \sqrt{3} = 2\sqrt{3}.$$

От правоъгълния ΔSCC_1

$$CC_1^2 = SC^2 + SC_1^2 = (2\sqrt{3})^2 + 2^2 = 16;$$

$$CC_1 = \sqrt{16} = 4;$$

$$V = \frac{h}{3}(B + B_1 + \sqrt{BB_1}) = \frac{2}{3} \left(\frac{9^2\sqrt{3}}{4} + \frac{3^2\sqrt{3}}{4} + \sqrt{\frac{9^2\sqrt{3}}{4} \cdot \frac{3^2\sqrt{3}}{4}} \right) =$$

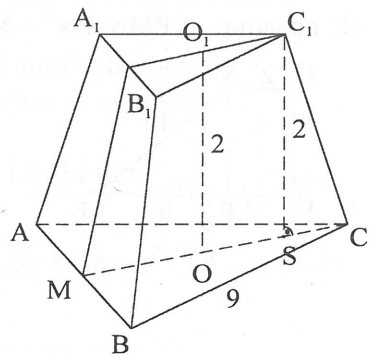
$$= \frac{2}{3} \left(\frac{81\sqrt{3}}{4} + \frac{9\sqrt{3}}{4} + \frac{27\sqrt{3}}{4} \right) = \frac{2}{3} \cdot \frac{117\sqrt{3}}{4} = \frac{39\sqrt{3}}{2}.$$

Стр.50, Зад.23. (Виж горния чертеж)

$$R = OC = \frac{a\sqrt{3}}{3} = \frac{13\sqrt{3}}{3}; R_1 = O_1C_1 = \frac{a\sqrt{3}}{3} = \frac{1\sqrt{3}}{3};$$

$$SC = R - R_1 = \frac{13\sqrt{3}}{3} - \frac{\sqrt{3}}{3} = 4\sqrt{3}; \text{ От правоъг. } \Delta SCC_1$$

$$CC_1^2 = SC^2 + SC_1^2; 8^2 = (4\sqrt{3})^2 + SC_1^2; SC_1^2 = 64 - 48 = 16;$$



$$SC = \sqrt{16} = 4 = h = OO_1; B = \frac{a^2\sqrt{3}}{4} = \frac{169\sqrt{3}}{4}; B_1 = \frac{1^2\sqrt{3}}{4};$$

$$V = \frac{h}{3}(B + B_1 + \sqrt{BB_1}) = \frac{4}{3} \left(\frac{169\sqrt{3}}{4} + \frac{1\sqrt{3}}{4} + \sqrt{\frac{169\sqrt{3}}{4} \cdot \frac{1\sqrt{3}}{4}} \right) =$$

$$= \frac{4}{3} \left(\frac{183}{4} \cdot \sqrt{3} \right) = 61\sqrt{3}.$$

Стр.50, Зад.24. От правоъгълния ΔSCC_1 $CC_1^2 = C_1S^2 + SC^2;$

$$5^2 = h^2 + 4^2; h^2 = 5^2 - 4^2 = 9; h = \sqrt{9} = 3.$$

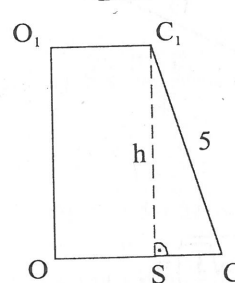
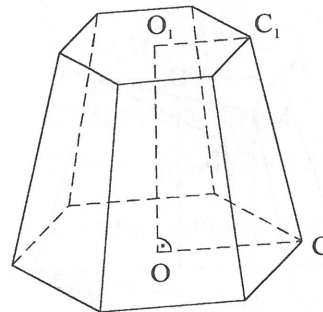
$$a = 6, R = OC = a = 6;$$

$$a_1 = 2, R_1 = O_1C_1 = a_1 = 2;$$

$$\ell = 5, SC = R - R_1 = 6 - 2 = 4.$$

$$B_1 = 6 \cdot \frac{2^2\sqrt{3}}{4} = 6\sqrt{3};$$

$$B = 6 \cdot \frac{6^2\sqrt{3}}{4} = 54\sqrt{3}$$



$$V = \frac{h}{3}(B + B_1 + \sqrt{BB_1}) =$$

$$= \frac{3}{3} \left(54\sqrt{3} + 6\sqrt{3} + \sqrt{54\sqrt{3} \cdot 6\sqrt{3}} \right) =$$

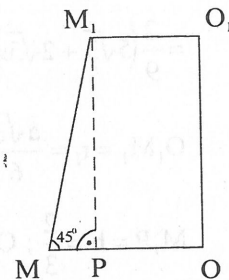
$$= 54\sqrt{3} + 6\sqrt{3} + 18\sqrt{3} = 78\sqrt{3}.$$

Стр.50, Зад.25. (Виж чертежа на зад.22)

$$\angle OMM_1 = 45^\circ;$$

$$a = 6; a_1 = 2; OM = r = \frac{a\sqrt{3}}{6} = \frac{6\sqrt{3}}{6} = \sqrt{3};$$

$$r_1 = O_1M_1 = \frac{a_1\sqrt{3}}{6} = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$$



$$MP = MO - M_1O_1 = \sqrt{3} - \frac{\sqrt{3}}{3} = \frac{2}{3}\sqrt{3};$$

От правоъг. ΔMPM_1 с $\angle PMM_1 = 45^\circ$ следва, че

$$MP = PM_1 = \frac{2}{3}\sqrt{3} = h; \quad B = \frac{a^2\sqrt{3}}{4} = \frac{6^2\sqrt{3}}{4} = 9\sqrt{3}; \quad B_1 = \frac{2^2\sqrt{3}}{4} = \sqrt{3};$$

$$V = \frac{h}{3}(B + B_1 + \sqrt{BB_1}) = \frac{2\sqrt{3}}{3.3}(9\sqrt{3} + \sqrt{3} + \sqrt{9\sqrt{3} \cdot \sqrt{3}}) = \\ = \frac{2\sqrt{3}}{9}(10\sqrt{3} + 3\sqrt{3}) = \frac{2\sqrt{3}}{9} \cdot 13\sqrt{3} = \frac{26}{3}.$$

Стр.51, Зад.26. $a = 4; a_1 = 2;$

$$R = \frac{a\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}; \quad R_1 = \frac{a_1\sqrt{3}}{3} = \frac{2\sqrt{3}}{3};$$

$$SC = R - R_1 = \frac{4\sqrt{3}}{3} - \frac{2\sqrt{3}}{3} = \frac{2\sqrt{3}}{3};$$

От правоъг. ΔSCC_1 , $SC_1 : SC = \text{tg} 30^\circ;$

$$SC_1 = SC \text{tg} 30^\circ = \frac{2\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{3} = \frac{2}{3} = h;$$

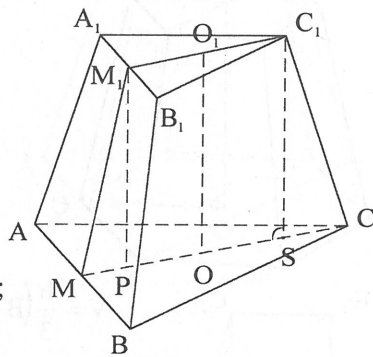
$$B = \frac{a^2\sqrt{3}}{4} = \frac{4^2\sqrt{3}}{4} = 4\sqrt{3}; \quad B_1 = \frac{2^2\sqrt{3}}{4} = \sqrt{3};$$

$$V = \frac{h}{3}(B + B_1 + \sqrt{BB_1}) = \frac{2}{3.3}(4\sqrt{3} + \sqrt{3} + \sqrt{4\sqrt{3} \cdot \sqrt{3}}) =$$

$$= \frac{2}{9}(5\sqrt{3} + 2\sqrt{3}) = \frac{2}{9} \cdot 7\sqrt{3} = \frac{14\sqrt{3}}{9}; \quad OM = r = \frac{a\sqrt{3}}{6} = \frac{4\sqrt{3}}{6} = \frac{2}{3}\sqrt{3};$$

$$O_1M_1 = r_1 = \frac{a_1\sqrt{3}}{6} = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}; \quad MP = r - r_1 = \frac{2}{3}\sqrt{3} - \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{3};$$

$$M_1P = h = \frac{2}{3}; \quad \text{От правоъгълния } \Delta MPM_1 \quad MM_1^2 = MP^2 + PM_1^2;$$



$$k = MM_1; \quad MM_1^2 = \left(\frac{\sqrt{3}}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = \frac{3}{9} + \frac{4}{9} = \frac{7}{9}; \quad MM_1 = \frac{\sqrt{7}}{3};$$

$$S = \frac{p+p_1}{2} \cdot k = \frac{3.4+3.2}{2} \cdot \frac{\sqrt{7}}{3} = 3\sqrt{7}.$$

Стр.51, Зад.27.

$$r = OM = \frac{a\sqrt{3}}{2} = \frac{8\sqrt{3}}{2} = 4\sqrt{3};$$

$$r_1 = O_1M_1 = \frac{a_1\sqrt{3}}{2} = \frac{4\sqrt{3}}{2} = 2\sqrt{3};$$

От правоъгълния ΔPMM_1 , където

$$PM = r - r_1 = 4\sqrt{3} - 2\sqrt{3} = 2\sqrt{3}, \quad PM_1 = h = 2,$$

$$k = MM_1^2 = PM^2 + PM_1^2 = (2\sqrt{3})^2 + 2^2 = 16;$$

$$MM_1 = 4;$$

$$\text{a) } S = \frac{p+p_1}{2} \cdot k = \frac{6.8+6.4}{2} \cdot 4 = \frac{48+24}{2} \cdot 4 = 144;$$

$$\text{б) } S_1 = S + 2B = 144 + 6 \cdot \frac{8^2\sqrt{3}}{4} + 6 \cdot \frac{4^2\sqrt{3}}{4} = 144 + 96\sqrt{3} + 24\sqrt{3} = \\ = 144 + 120\sqrt{3};$$

в) От правоъгълния ΔPMM_1 $PM_1 : PM = \text{tg} \alpha;$

$$\text{tg} \alpha = 2 : 2\sqrt{3} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}; \quad \text{следователно } \alpha = 30^\circ.$$

