$a_1 = -30 - 10.(-3,5) = -30 + 35 = 5$;

в) $\begin{vmatrix} a_7 = 0 \\ a_{30} = 23\sqrt{2} \end{vmatrix} \rightarrow - \begin{vmatrix} a_1 + 6d = 0 \\ a_1 + 29d = 23\sqrt{2} \end{vmatrix}$; $-23d = -23\sqrt{2}$; $d = \sqrt{2}$;

$a_1 + 6\sqrt{2} = 0$; $a_1 = -6\sqrt{2}$.

Стр.50, Зад.6.

а) $\begin{vmatrix} a_4 = 27 \\ a_6 = 243 \end{vmatrix} \rightarrow : \begin{vmatrix} a_1 q^3 = 27 \\ a_1 q^5 = 243 \end{vmatrix} \rightarrow \frac{1}{q^2} = \frac{1}{9}$; $q^2 = 9$; $q = \pm 3$; $a_1 = 27 : 3^3 = 1$;

$a_1^{|} = 27 : (-27) = -1$; $a_5 = a_1 q^4 = 1.3^4 = 81$; $a_5^{|} = -1.(-3)^4 = -81$;

б) $\begin{vmatrix} a_3 = 0,04 \\ a_7 = 0,000004 \end{vmatrix} \rightarrow : \begin{vmatrix} a_1 q^2 = 0,04 \\ a_1 q^6 = 0,000004 \end{vmatrix}$; $\frac{1}{q^4} = \frac{0,04}{0,000004} = \frac{40000}{4} = 10000$;

$q^4 = \frac{1}{10000}$; $q = \frac{1}{10}$; $a_1 \frac{1}{10^2} = 0,04$; $a_1 = 0,04.10^2 = 4$;

$a_5 = a_1 q^4 = 4.\frac{1}{10^4} = 0,0004$;

в) $\begin{vmatrix} a_2 = 8 \\ a_9 = 64\sqrt{2} \end{vmatrix} \rightarrow : \begin{vmatrix} a_1 q = 8 \\ a_1 q^8 = 64\sqrt{2} \end{vmatrix} \rightarrow \frac{1}{q^7} = \frac{1}{8\sqrt{2}}$;

$q^7 = 8\sqrt{2} = 2^{3+\frac{1}{2}} = 2^{\frac{7}{2}} = \left(\sqrt{2}\right)^7$; $q = \sqrt{2}$; $a_1\sqrt{2} = 8$; $a_1 = \frac{8}{\sqrt{2}} = 4\sqrt{2}$;

$a_5 = a_1 q^4 = 4\sqrt{2}.\left(\sqrt{2}\right)^4 = 4\sqrt{2}.4 = 16\sqrt{2}$.

Стр.50, Зад.7. а) $a_1 = 5$; $d = 3$; $n = 10$;

$S_{10} = \frac{2a_1 + (n-1)d}{2}.10 = \frac{2.5 + (10-1)3}{2}.10 = \frac{10+27}{2}.10 = 37.5 = 185$;

б) $a_n = 0,5$; $d = -1,5$; $n = 15$;

$S_{15} = \frac{2a_n + (n-1)d}{2}.15 = \frac{2.0,5 + (15-1)(-1,5)}{2}.15 = \frac{1-21}{2}.15 = -150$;

в) $S_{31} = ?$; $a_1 = 15\sqrt{2}$; $d = -\sqrt{2}$;

$S_{31} = \frac{2.15\sqrt{2} + (31-1)(-\sqrt{2})}{2}.31 = \frac{30\sqrt{2} - 30\sqrt{2}}{2}.31 = 0$.

Стр.50, Зад.8. а) $S_{10} = \frac{a_1 + a_n}{2}.n = \frac{7+23}{2}.10 = 15.10 = 150$;

б) $S_{15} = \frac{(-2)+(-26)}{2}.15 = -\frac{28}{2}.15 = -14.15 = -210$;

в) $S_{21} = ?$; $a_1 = 0,5$; $a_{21} = 3,5$; $S_{21} = \frac{a_1 + a_n}{2}.n = \frac{0,5+3,5}{2}.21 = 42$.

Стр.50, Зад.9. а) $a_1 = 8$; $q = 0,5$; $n = 5$;

$S_5 = a_1 \frac{q^5 - 1}{q - 1} = 8.\frac{\left(\frac{1}{2}\right)^5 - 1}{\frac{1}{2} - 1} = 8.\frac{\frac{1}{32} - 1}{-\frac{1}{2}} = \left(\frac{1}{4} - 8\right)(-2) = 16 - \frac{1}{2} = 15,5$;

б) $\begin{vmatrix} a_4 = -8 \\ a_{10} = -512 \end{vmatrix} \rightarrow : \begin{vmatrix} a_1 q^3 = -8 \\ a_1 q^9 = -512 \end{vmatrix}$; $\frac{1}{q^6} = \frac{1}{64}$; $q^6 = 64 = 2^6$; $q = 2$;

$a_1 2^3 = -8$; $a_1 = -1$; $S_5 = a_1 \frac{q^n - 1}{q - 1} = -1.\frac{2^5 - 1}{2 - 1} = -31$;

в) $a_1 = \sqrt{2} - 1$; $a_5 = 4\sqrt{2} - 4$; $a_5 = a_1 q^4 = \left(\sqrt{2} - 1\right)q^4 = 4\sqrt{2} - 4$;

$q^4 = \frac{4\sqrt{2} - 4}{\sqrt{2} - 1} = 4$; $q^4 = \left(\sqrt{2}\right)^4$; $q = \pm\sqrt{2}$;

$S_{12} = a_1 \frac{q^{12} - 1}{q - 1} = \left(\sqrt{2} - 1\right)\frac{\left(\sqrt{2}\right)^{12} - 1}{\sqrt{2} - 1} = \sqrt{2^{12}} - 1 = 2^6 - 1 = 63$.

Стр.50, Зад.10. а) $\div a_1 = 2$, $S_{10} = 245$; $S_{10} = \frac{a_1 + a_{10}}{2}.10 = 245$;

$2 + a_{10} = 49$; $a_{10} = 47$;

б) $S_{17} = \frac{2a_1 + (17-1)(-4)}{2}.17 = -272$; $2a_1 - 64 = -32$; $2a_1 = 32$; $a_1 = 16$;

$a_{10} = a_1 + 9d = 16 + 9(-4) = 16 - 36 = -20$;

в) $a_1=-33$; $S_{19}=0$; $S_{19}=\frac{2a_1+(19-1)d}{2}.19=0$; $2a_1+18d=0$;

$2(-33)+18d=0$; $18d=66$; $3d=11$; $d=\frac{11}{3}$;

$a_{10}=a_1+9d=-33+9.\frac{11}{3}=-33+33=0$.

Стр.50, Зад.11.

а) $a_1=3$; $d=-2$; $S_n=-165$; $S_n=\frac{2a_1+(n-1)d}{2}.n=-165$;

$\frac{2.3+(n-1)(-2)}{2}.n=-165$; $(3-n+1)n=-165$; $4n-n^2+165=0$;

$n^2-4n-165=0$; $n_{1,2}=2\pm\sqrt{4+165}=2\pm13$; $n_1=15$; $n_2=-11$ - не е решение;

б) $a_6=-2{,}5$; $d=0{,}5$; $S_n=-17$; $a_6=a_1+5d=a_1+5.0{,}5=a_1+2{,}5=-2{,}5$;

$a_1=-5$; $S_n=\frac{2a_1+(n-1)d}{2}.n=-17$; $\frac{2(-5)+(n-1)0{,}5}{2}.n=-17$;

$(-10+0{,}5n-0{,}5)n=-34\ |.2$; $(-20+n-1)n=-68$; $n^2-21n+68=0$;

$n_{1,2}=\frac{21\pm\sqrt{441-272}}{2}=\frac{21\pm13}{2}$; $n_1=17$; $n_2=4$;

в) $a_7=-4$; $a_{11}=0$; $S_n=-19$; $-\begin{vmatrix}a_1+6d=-4\\a_1+10d=0\end{vmatrix}\rightarrow 4d=4;\ d=1$;

$a_1+6.1=-4$; $a_1=-10$; $S_n=\frac{2a_1+(n-1)d}{2}.n=-19$;

$\frac{2(-10)+(n-1).1}{2}.n=-19$; $(-20+n-1)n=-38$; $n^2-21n+38=0$;

$n_{1,2}=\frac{21\pm\sqrt{441-152}}{2}=\frac{21\pm17}{2}$; $n_1=19$; $n_2=2$.

Стр.50, Зад.12.

$\begin{vmatrix}a_5-a_3=288\\a_6-a_4=864\end{vmatrix}$; $\begin{vmatrix}a_1q^4-a_1q^2=288\\a_1q^5-a_1q^3=864\end{vmatrix}$; : $\begin{vmatrix}a_1q^2(q^2-1)=288\\a_1q^3(q^2-1)=864\end{vmatrix}$; $\frac{1}{q}=\frac{1}{3}$; $q=3$.

Стр.51, Зад.13. а) $a_1=1$; $q=3$; $S_n=121$; $S_n=a_1\frac{q^n-1}{q-1}=121$;

$\frac{3^n-1}{3-1}=121$; $3^n-1=242$; $3^n=243$; $3^n=3^5$; $n=5$;

б) $a_1=64$; $q=-\frac{1}{2}$; $S_n=42$; $42=a_1\frac{q^n-1}{q-1}$; $64.\frac{\left(-\frac{1}{2}\right)^n-1}{-\frac{1}{2}-1}=42$;

$64.\frac{\left(-\frac{1}{2}\right)^n-1}{-\frac{3}{2}}=42$; $64\left[\left(-\frac{1}{2}\right)^n-1\right]=-63$; $\left(-\frac{1}{2}\right)^n-1=-\frac{63}{64}$;

$\left(-\frac{1}{2}\right)^n=-\frac{63}{64}+1$; $\left(-\frac{1}{2}\right)^n=\frac{1}{64}$; $\left(-\frac{1}{2}\right)^n=\left(-\frac{1}{2}\right)^6$; $n=6$.

Стр.51, Зад.14.

$2+4+6+...+2n=n(n+1)$?; $2+4+6+...+2n$ е ÷ с $a_1=2$, $d=4-2$;

$n=n$, следователно $S=\frac{2+2n}{2}.n=(1+n)n$.

Стр.51, Зад.15. Нека времето е n и търсения срок е $8n$ и нека количеството означим с Q. Ще имаме ÷ с $a_1=Q$, $q=\frac{1}{2}$, $a_n=0{,}001$; $a_n=a_1q^{n-1}$ или

$0{,}001.Q=Q.\left(\frac{1}{2}\right)^{n-1}$ или $0{,}001=\left(\frac{1}{2}\right)^{n-1}$; $\frac{1}{2^{n-1}}=0{,}001$; $0{,}001.2^{n-1}=1$;

$\frac{1}{1000}.2^{n-1}=1$; $2^{n-1}=1000$; $(n-1)\lg 2=\lg 1000$; $n-1=\frac{\lg 1000}{\lg 2}=\frac{3}{0{,}3010}$;

$n=\frac{3}{0{,}3010}+1=\frac{3000}{301}+1\approx 11$ дни;

$11.8\approx 88$ дни. След 88 дни ще остане по-малко от 0,001 от даденото количество.

Стр.51, Зад.16. Броят на участниците в отделните кръгове образуват геометрична прогресия, за която $a_8=2$; $a_7=4$; $q=\frac{a_8}{a_7}=\frac{2}{4}=\frac{1}{2}$;

$a_8=a_1.q^7=2$; $a_1.\left(\frac{1}{2}\right)^7=2$; $a_1.\frac{1}{2^7}=2$; $a_1=2^8=256$ - броя на всички участници.