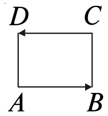
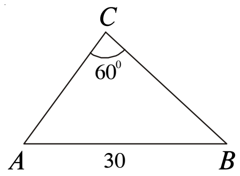
Стр.210 / Тест 5



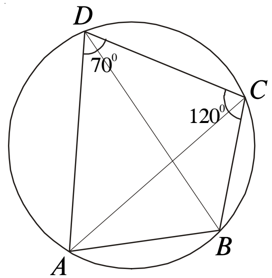
Стр.210, Зад.1. в) $\overrightarrow{AB}.\overrightarrow{CD} = \left|\overrightarrow{AB}\right|.\left|\overrightarrow{CD}\right|.\cos 180^0 = 1.1.(-1) = -1.$

Стр.210, Зад.2. $AB = 30$; $\measuredangle C = 60^0$; $R = ?$



От синусова теорема следва, че $\dfrac{AB}{\sin 60^0} = 2R$ или

$\dfrac{30}{\sin 60^0} = 2R$; $\dfrac{15}{\frac{\sqrt{3}}{2}} = R$; $R = \dfrac{30}{\sqrt{3}}$; $R = 10\sqrt{3}$.



Стр.210, Зад.3. $ABCD$ - вписан четириъгълник, $R = 40$; $\measuredangle C = 120^0$; $\measuredangle D = 70^0$.

От синусова теорема за ΔBCD следва, че

$\dfrac{BD}{\sin \measuredangle C} = 2R$ или $\dfrac{BD}{\sin 120^0} = 2.40$;

$BD = 80.\sin 120^0 = 80.\dfrac{\sqrt{3}}{2} = 40\sqrt{3}$.

Стр.210, Зад.4. $a = 3$; $b = 6$; $c = 4$ $\Rightarrow$ $a^2 = 9$; $b^2 = 36$; $c^2 = 8$;

Тогава $36 > 9 + 8$ или $b^2 > a^2 + c^2$. Следователно ΔABC е тъпоъгълен с тъп ъгъл при върха B.

Стр.210, Зад.5. $BC = a$; $AC = b$; $\measuredangle C = 120^0$; $AB = ?$

От косинусова теорема следва, че $AB^2 = BC^2 + AC^2 - 2.BC.AC.\cos \measuredangle C$;

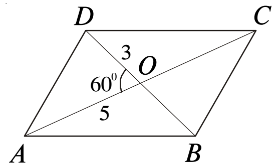
$AB = \sqrt{a^2 + b^2 - 2.a.b.\cos 120^0} = \sqrt{a^2 + b^2 - 2.a.b.\left(-\dfrac{1}{2}\right)} = \sqrt{a^2 + b^2 + a.b}$.

Стр.210, Зад.6.

От синусова теорема за ΔABC следва, че $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma} \Rightarrow a = \dfrac{c.\sin\alpha}{\sin\gamma}$;

$a = \dfrac{c.\sin\alpha}{\sin\gamma} = \dfrac{c.\sin\alpha}{\sin\left[180^0 - (\alpha+\beta)\right]} = \dfrac{c.\sin\alpha}{\sin(\alpha+\beta)}$.

Стр.210, Зад.7. $ABCD$ - успоредник, $AC = 10$; $BD = 6$; $\measuredangle AOD = 60^0$;



$AO = \dfrac{AC}{2} = 5$; $DO = \dfrac{BD}{2} = 3$.

От косинусова теорема за ΔAOD следва, че

$AD^2 = AO^2 + DO^2 - 2.AO.DO.\cos \measuredangle AOD$;

$AD^2 = 5^2 + 3^2 - 2.5.3.\cos 60^0 = 25 + 9 - 30.\dfrac{1}{2} = 34 - 15 = 19$; $AD = \sqrt{19}$.

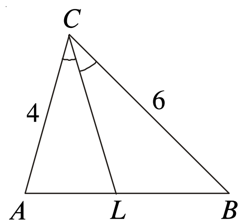
Стр.210, Зад.8. $a = 24$; $b = 26$; $c = 10$; $m_b = ?$

От формулата за медиана $m_b = \dfrac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}$ следва, че

$m_b = \dfrac{1}{2}\sqrt{2.24^2 + 2.10^2 - 26^2} = \dfrac{1}{2}\sqrt{1152 + 200 - 676} = \dfrac{1}{2}\sqrt{676} = \dfrac{26}{2} = 13$.

Стр.210, Зад.9. $r = 8$; $R = 18$. От формулата на Ойлер $d = \sqrt{R(R-2r)}$ следва, че $d = \sqrt{18(18 - 2.8)} = \sqrt{18.2} = \sqrt{36} = 6$.

Стр.210, Зад.10. От CL - ъглополовяща следва, че $\dfrac{AL}{BL} = \dfrac{AC}{BC}$ или



$\dfrac{AL}{BL} = \dfrac{4}{6}$; $\dfrac{AL}{BL} = \dfrac{2}{3}$; $\dfrac{AL + BL}{BL} = \dfrac{2+3}{3}$; $\dfrac{AB}{BL} = \dfrac{5}{3}$;

$\dfrac{5}{BL} = \dfrac{5}{3} \Rightarrow BL = 3$; $AL = AB - BL = 5 - 3 = 2$;

От формулата за ъглополовяща $\ell_c^{\,2} = a.b - m.n$ следва, че $CL^2 = AC.BC - AL.BL = 4.6 - 2.3 = 24 - 6 = 18$; $CL = \sqrt{18} = 3\sqrt{2}$.

Стр.211 / Контролна работа 9

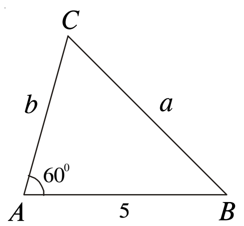
Стр.211, Зад.1. $b.\cos\gamma - c.\cos\beta = \dfrac{b^2 - c^2}{2R.\sin\alpha}$?

От синусова теорема за ΔABC следва, че $\frac{a}{\sin\alpha}=2R$, откъдето $a=2R.\sin\alpha$. От косинусова теорема за страната b следва, че 1) $b^2=a^2+c^2-2.a.c.\cos\beta$ и от косинусова теорема за страната c следва, че 2) $c^2=a^2+b^2-2.a.b.\cos\gamma$. Като извадим 1) и 2) получаваме:

$b^2-c^2=c^2-b^2-2.a.c.\cos\beta+2.a.b.\cos\gamma$; $2\left(b^2-c^2\right)=2\left(a.b.\cos\gamma-a.c.\cos\beta\right)$;

$b^2-c^2=a.b.\cos\gamma-a.c.\cos\beta$; $\frac{b^2-c^2}{a}=b.\cos\gamma-c.\cos\beta$; Но $a=2R.\sin\alpha$,

следователно $\frac{b^2-c^2}{2R.\sin\alpha}=b.\cos\gamma-c.\cos\beta$.



Стр.211, Зад.2. $c=5$; $\alpha=60^0$; $R=\frac{7\sqrt{3}}{3}$.

От синусова теорема следва, че $\frac{a}{\sin\alpha}=2R$ или

$\frac{a}{\sin 60^0}=2.\frac{7\sqrt{3}}{3}$; $a=\frac{14\sqrt{3}}{3}.\sin 60^0=\frac{14\sqrt{3}}{3}.\frac{\sqrt{3}}{2}=7$;

От косинусова теорема следва, че $a^2=c^2+b^2-2.c.b.\cos\alpha$ или

$7^2=5^2+b^2-2.5.b.\cos 60^0$; $49=25+b^2-10.b.\frac{1}{2}$; $b^2-5b-24=0$;

$D=25+96=121$; $b=\frac{5+11}{2}=8$.

Стр.211, Зад.3. $c=3$; $a=5$; $m_b=\frac{\sqrt{19}}{2}$; $b=?$ $m_a=?$ $\sin\gamma=?$ $R=?$

От формулата за медиана $m_b=\frac{1}{2}\sqrt{2a^2+2c^2-b^2}$ следва, че

$\frac{\sqrt{19}}{2}=\frac{1}{2}\sqrt{2.5^2+2.3^2-b^2}$; $\left(\sqrt{19}\right)^2=\left(\sqrt{50+18-b^2}\right)^2$; $19=68-b^2$;

$b^2=49$; $b=7$; $m_a=\frac{1}{2}\sqrt{2b^2+2c^2-a^2}=\frac{1}{2}\sqrt{2.7^2+2.3^2-5^2}=$

$=\frac{1}{2}\sqrt{98+18-25}=\frac{\sqrt{91}}{2}$; От косинусова теорема за страната c следва,

че $c^2=a^2+b^2-2.a.b.\cos\gamma$, откъдето $\cos\gamma=\frac{a^2+b^2-c^2}{2.a.b}$ или

$\cos\gamma=\frac{5^2+7^2-3^2}{2.5.7}=\frac{25+49-9}{70}=\frac{65}{70}$; $\sin\gamma=\sqrt{1-\cos^2\gamma}=\sqrt{1-\left(\frac{65}{70}\right)^2}=$

$=\sqrt{1-\frac{4225}{4900}}=\sqrt{\frac{4900-4225}{4900}}=\sqrt{\frac{675}{4900}}=\frac{15\sqrt{3}}{70}=\frac{3\sqrt{3}}{14}$;

От синусова теорема $\frac{c}{\sin\gamma}=2R$ следва, че $R=\frac{c}{2.\sin\gamma}$ или

$R=\frac{3}{2.\frac{3\sqrt{3}}{14}}=\frac{7}{\sqrt{3}}.\frac{\sqrt{3}}{\sqrt{3}}=\frac{7\sqrt{3}}{3}$.

Стр.211 / Контролна работа 10

Стр.211, Зад.1. $\frac{a^2-b^2}{c^2}=a.\cos\beta-b.\cos\alpha$?

От косинусова теорема за страната a следва 1) $a^2=b^2+c^2-2.b.c.\cos\alpha$ и от косинусова теорема за страната b - 2) $b^2=a^2+c^2-2.a.c.\cos\beta$.
Като извадим 1) и 2) получаваме:

$a^2-b^2=b^2-a^2-2.b.c.\cos\alpha+2.a.c.\cos\beta$;

$2.a^2-2.b^2=2.a.c.\cos\beta-2.b.c.\cos\alpha$;

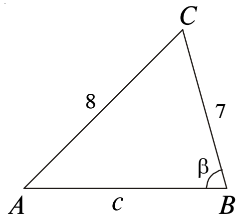
$a^2-b^2=a.c.\cos\beta-b.c.\cos\alpha$; $a^2-b^2=c.\left(a.\cos\beta-b.\cos\alpha\right)$;

$\frac{a^2-b^2}{c^2}=a.\cos\beta-b.\cos\alpha$.

Стр.211, Зад.2. $a = 7$; $b = 8$; $\cos\beta = \dfrac{1}{7}$; $\alpha = ?$ $c = ?$ $R = ?$

$$\sin\beta = \sqrt{1-\cos^2\beta} = \sqrt{1-\left(\frac{1}{7}\right)^2} = \sqrt{1-\frac{1}{49}} = \sqrt{\frac{49-1}{49}} = \sqrt{\frac{48}{49}} = \frac{4\sqrt{3}}{7};$$

От синусова теорема $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$ следва, че $\sin\alpha = \dfrac{a.\sin\beta}{b}$ или



$$\sin\alpha = \frac{7.\frac{4\sqrt{3}}{7}}{8} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 60^0;$$

От косинусова теорема следва, че $a^2 = b^2 + c^2 - 2.b.c.\cos\alpha$ или

$49 = 64 + c^2 - 2.8.c.\cos 60^0$; $c^2 - 8.c + 15 = 0$; $D = 16 - 15 = 1$; $c = 4 + 1 = 5$;

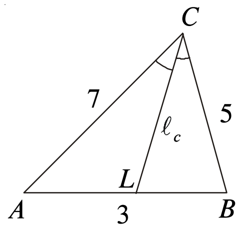
От синусова теорема $\dfrac{a}{\sin\alpha} = 2R \Rightarrow R = \dfrac{a}{2.\sin\alpha} = \dfrac{7}{2.\frac{\sqrt{3}}{2}} = \dfrac{7}{\sqrt{3}}.\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{7\sqrt{3}}{3}$.

Стр.211, Зад.3. $c = 3$; $b = 7$; $m_a = \dfrac{\sqrt{91}}{2}$; $a = ?$ $\ell_c = ?$ $\beta = ?$ $r = ?$

От формулата за медиана $m_a = \dfrac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$ следва, че

$m_a{}^2 = \dfrac{1}{4}\left(2b^2 + 2c^2 - a^2\right)$ или $\dfrac{91}{4} = \dfrac{1}{4}\left(2.7^2 + 2.3^2 - a^2\right)$; $91 = 98 + 18 - a^2$;

$a^2 = 25$; $a = 5$; От CL - ъглополовяща следва, че $\dfrac{AL}{LB} = \dfrac{AC}{BC}$ или



$\dfrac{AL}{LB} = \dfrac{7}{5}$; $\dfrac{AL+BL}{LB} = \dfrac{7+5}{5}$; $\dfrac{AB}{LB} = \dfrac{12}{5}$; $\dfrac{3}{LB} = \dfrac{12}{5}$;

$LB = \dfrac{15}{12}$; $LB = \dfrac{5}{4}$; $AL = AB - LB = 3 - \dfrac{5}{4} = \dfrac{7}{4}$;

От формулата $\ell_c{}^2 = AC.BL - AL.LB$ следва, че

$\ell_c{}^2 = 7.5 - \dfrac{7}{4}.\dfrac{5}{4} = 35 - \dfrac{35}{16} = \dfrac{525}{16}$; $\ell_c = \sqrt{\dfrac{525}{16}} = \dfrac{5\sqrt{21}}{4}$;

От косинусова теорема $b^2 = a^2 + c^2 - 2.a.c.\cos\beta$ следва, че

$\cos\beta = \dfrac{a^2 + c^2 - b^2}{2.a.c}$ или $\cos\beta = \dfrac{25+9-49}{2.5.3} = \dfrac{-15}{30} = -\dfrac{1}{2} \Rightarrow \beta = 120^0$.

$S_\Delta = \dfrac{a.c.\sin\beta}{2} = \dfrac{5.3.\sin 120^0}{2} = \dfrac{15}{2}.\dfrac{\sqrt{3}}{2} = \dfrac{15\sqrt{3}}{4}$;

Но $S_\Delta = p.r = \dfrac{a+b+c}{2}.r = \dfrac{5+7+3}{2}.r = \dfrac{15}{2}.r \Rightarrow \dfrac{15}{2}.r = \dfrac{15\sqrt{3}}{4}$; $r = \dfrac{\sqrt{3}}{2}$.

Стр.219, Зад.1. $S_{\Delta ABC} = \dfrac{a.b}{2}.\sin\gamma = \dfrac{6.4\sqrt{3}}{2}.\sin 120^0 = 12\sqrt{3}.\dfrac{\sqrt{3}}{2} = 6.3 = 18.$

Стр.219, Зад.2. $S_{\Delta ABC} = \dfrac{b.c}{2}.\sin\alpha = \dfrac{3\sqrt{7}.8}{2}.\sin 30^0 = 12\sqrt{7}.\dfrac{1}{2} = 6\sqrt{7}.$

Стр.219, Зад.3. $S_{\Delta ABC} = \dfrac{a.c}{2}.\sin\beta = \dfrac{10.6\sqrt{2}}{2}.\sin 135^0 =$

$= 30\sqrt{2}.\sin\left(180^0 - 45^0\right) = 30\sqrt{2}.\sin 45^0 = 30\sqrt{2}.\dfrac{\sqrt{2}}{2} = 30.$

Стр.219, Зад.4. $a = 4\sqrt{7}$, $c = 4$, $\measuredangle\alpha = 120^0$, $S_{\Delta ABC} = ?$

От косинусова теорема за страната a следва, че $a^2 = b^2 + c^2 - 2bc.\cos\alpha$;

$\left(4\sqrt{7}\right)^2 = b^2 + 4^2 - 2.b.4.\cos 120^0$; $112 = b^2 + 16 - 8b.\left(-\dfrac{1}{2}\right)$;

$b^2 + 4b - 96 = 0$; $b = -2 + \sqrt{4+96} = -2 + 10 = 8$;

$S_{\Delta ABC} = \dfrac{b.c}{2}.\sin\alpha = \dfrac{8.4}{2}.\sin 120^0 = 16.\dfrac{\sqrt{3}}{2} = 8\sqrt{3}.$

Стр.219, Зад.5. $a = 4$, $\measuredangle\beta = 30^0$, $\measuredangle\gamma = 45^0$; $S_{\Delta ABC} = ?$

$\measuredangle\alpha = 180^0 - (\measuredangle\beta + \measuredangle\gamma) = 180^0 - \left(30^0 + 45^0\right) = 180^0 - 75^0 = 105^0 = 90^0 + 15^0$;

$$\sin\measuredangle\alpha=\sin\left(90^0+15^0\right)=\cos15^0=\frac{\sqrt{6}+\sqrt{2}}{4};$$

$$S_{\Delta ABC}=\frac{a^2.\sin\beta.\sin\gamma}{2\sin\alpha}=\frac{4^2.\sin30^0.\sin45^0}{2.\frac{\sqrt{6}+\sqrt{2}}{4}}=\frac{2.16.\frac{1}{2}.\frac{\sqrt{2}}{2}}{\sqrt{6}+\sqrt{2}}=$$

$$=\frac{8\sqrt{2}}{\sqrt{6}+\sqrt{2}}.\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}=\frac{8\left(2\sqrt{3}-2\right)}{6-2}=\frac{16\left(\sqrt{3}-1\right)}{4}=4\left(\sqrt{3}-1\right).$$

Стр.219, Зад.6. $S_{\Delta ABC}=?$

$$p=\frac{a+b+c}{2}=\frac{9+6+5}{2}=\frac{20}{2}=10;$$

$$p-a=10-9=1;\; p-b=10-6=4;\; p-c=10-5=5;$$

$$S_{\Delta ABC}=\sqrt{p(p-a)(p-b)(p-c)}=\sqrt{10.1.4.5}=2\sqrt{50}=2.5\sqrt{2}=10\sqrt{2}.$$

Стр.219, Зад.7. $S_{\Delta ABC}=?$

$$p=\frac{a+b+c}{2}=\frac{9+10+17}{2}=\frac{36}{2}=18;$$

$$p-a=18-9=9;\; p-b=18-10=8;\; p-c=18-17=1;$$

$$S_{\Delta ABC}=\sqrt{p(p-a)(p-b)(p-c)}=\sqrt{18.9.8.1}=3\sqrt{144}=3.12=36.$$

Стр.219, Зад.8. $S_{\Delta ABC}=?$

$$p=\frac{a+b+c}{2}=\frac{8+26+30}{2}=\frac{64}{2}=32;$$

$$p-a=32-8=24;\; p-b=32-26=6;\; p-c=32-39=2;$$

$$S_{\Delta ABC}=\sqrt{p(p-a)(p-b)(p-c)}=\sqrt{32.24.6.2}=\sqrt{64.4.6.6}=8.2.6=96.$$

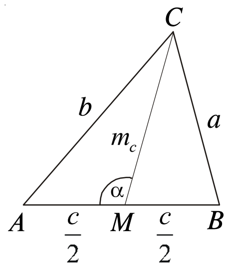
Стр.219, Зад.9. $S_{\Delta ABC}=?$

$$p=\frac{a+b+c}{2}=\frac{26+75+91}{2}=\frac{64}{2}=96;$$

$$p-a=96-26=70;\; p-b=96-75=21;\; p-c=96-91=5;$$

$$S_{\Delta ABC}=\sqrt{p(p-a)(p-b)(p-c)}=\sqrt{96.70.21.5}=\sqrt{2.3.16.7.2.5.3.7.5}=$$

$$=\sqrt{2^2.3^2.4^2.5^2.7^2}=2.3.4.5.7=10.12.7=840.$$



Стр.219, Зад.10.

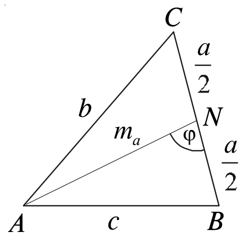
От CM - медиана, следва че $AM=MB=\frac{AB}{2}=\frac{c}{2}$.

Нека $\measuredangle AMC=\alpha$. Тогава $\measuredangle BMC=180^0-\alpha$.

$$S_{\Delta AMC}=\frac{AM.CM}{2}.\sin\measuredangle AMC=\frac{c}{2}.\frac{m_c}{2}.\sin\measuredangle AMC=\frac{c.m_c}{4}.\sin\alpha;$$

$$S_{\Delta BMC}=\frac{MB.CM}{2}.\sin\measuredangle BMC=\frac{c}{2}.\frac{m_c}{2}.\sin\left(180^0-\alpha\right)=\frac{c.m_c}{4}.\sin\alpha;$$

Следователно $S_{\Delta AMC}=S_{\Delta BMC}$.



От AN - медиана, следва че $BN=NC=\frac{BC}{2}=\frac{a}{2}$;

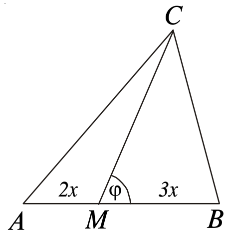
Нека $\measuredangle ANB=\alpha$. Тогава $\measuredangle ANC=180^0-\varphi$.

$$S_{\Delta ANB}=\frac{AN.BN}{2}.\sin\measuredangle ANB=\frac{m_a}{2}.\frac{a}{2}.\sin\alpha=\frac{a.m_a}{4}.\sin\alpha;$$

$$S_{\Delta ANC}=\frac{AN.NC}{2}.\sin\measuredangle ANC=\frac{m_a}{2}.\frac{a}{2}.\sin\left(180^0-\alpha\right)=\frac{a.m_a}{4}.\sin\alpha;$$

Следователно $S_{\Delta ANB} = S_{\Delta ANC}$.

По аналогия се доказва, че и m_b разделя ΔABC на два равнолицеви триъгълника.



Стр.219, Зад.11. Ако $\dfrac{AM}{MB} = \dfrac{2}{3}$, то $\dfrac{S_{\Delta AMC}}{S_{\Delta MBC}} = \dfrac{2}{3}$?

Нека $AM = 2x$. Тогава $MB = 3x$. Нека $\measuredangle BMC = \varphi$.

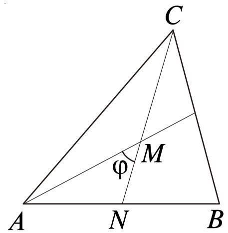
Тогава $\measuredangle AMC = 180^0 - \measuredangle BMC = 180^0 - \varphi$.

$$S_{\Delta BMC} = \frac{BM.CM}{2}.\sin\measuredangle BMC = \frac{3x.CM}{2}.\sin\varphi;$$

$$S_{\Delta AMC} = \frac{AM.CM}{2}.\sin\measuredangle AMC = \frac{2x.CM}{2}.\sin\left(180^0 - \varphi\right) = \frac{2x.CM}{2}.\sin\varphi;$$

$$\frac{S_{\Delta AMC}}{S_{\Delta MBC}} = \frac{\frac{2x.CM}{2}.\sin\varphi}{\frac{3x.CM}{2}.\sin\varphi} = \frac{2}{3}.$$

Стр.219, Зад.12. $\dfrac{S_{\Delta AMN}}{S_{\Delta AMC}} = \dfrac{1}{2}$?



От т.M - медицентър, следва че $\dfrac{CM}{MN} = \dfrac{2}{1}$.

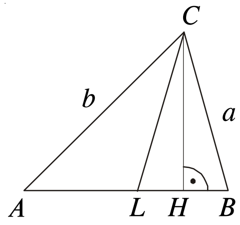
Нека $MN = x$. Тогава $CM = 2x$. Нека $\measuredangle AMN = \varphi$.

Тогава $\measuredangle AMC = 180^0 - \varphi$.

$$S_{\Delta AMN} = \frac{AM.MN}{2}.\sin\measuredangle AMN = \frac{AM.x}{2}.\sin\varphi;$$

$$S_{\Delta AMC} = \frac{AM.CM}{2}.\sin\measuredangle AMC = \frac{AM.2x}{2}.\sin\left(180^0 - \alpha\right) = \frac{AM.2x}{2}.\sin\alpha;$$

$$\frac{S_{\Delta AMN}}{S_{\Delta AMC}} = \frac{\frac{AM.x}{2}.\sin\varphi}{\frac{AM.2x}{2}.\sin\alpha} = \frac{1}{2}.$$



Стр.219, Зад.13. $S_{\Delta ABC} = S$; $BC = a$; $AC = b$.

Нека $AB = c$ и височината $CH = h_c$.

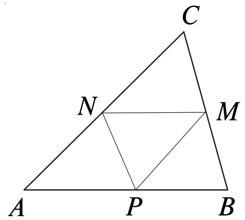
От CL - ъглополовяща следва, че

$$\frac{AL}{BL} = \frac{AC}{BC};\ \frac{AL}{BL} = \frac{b}{a};\ \frac{AL+BL}{BL} = \frac{a+b}{a};\ \frac{c}{BL} = \frac{a+b}{a};$$

$$BL = \frac{a.c}{a+b};\ AL = AB - BL = c - \frac{a.c}{a+b} = \frac{c.a + b.c - a.c}{a+b} = \frac{b.c}{a+b};$$

$$S_{\Delta ALC} = \frac{AL.h_c}{2} = \frac{b.c}{a+b}.\frac{h_c}{2},\ \text{но}\ S_{\Delta ABC} = \frac{c.h_c}{2} \Rightarrow S_{\Delta ALC} = \frac{b}{a+b}.\frac{c.h_c}{2} = \frac{b.S}{a+b}.$$

По аналогия $S_{\Delta BLC} = \dfrac{BL.h_c}{2} = \dfrac{a.c}{a+b}.\dfrac{h_c}{2} = \dfrac{a.S}{a+b}$.



Стр.219, Зад.14. $S_{\Delta ABC} = S$; M - среда на BC; N - среда на AC и P - среда на AB.

От M - среда на BC и N - среда на AC следва, че MN е средна отсечка в ΔABC, следователно $MN \parallel AB$ и $MN = \dfrac{AB}{2}$. По аналогия доказваме, че $MP = \dfrac{AC}{2}$ и $MP \parallel AC$ и $NP = \dfrac{BC}{2}$ и $NP \parallel BC$. Следователно

$\Delta MNP \sim \Delta ABC$ по III признак: $\dfrac{MN}{AB} = \dfrac{NP}{BC} = \dfrac{MP}{AC} = \dfrac{1}{2} = k$.

Следователно $\dfrac{S_{\Delta MNP}}{S_{\Delta ABC}} = k^2$ или $\dfrac{S_{\Delta MNP}}{S} = \left(\dfrac{1}{2}\right)^2$; $S_{\Delta MNP} = \dfrac{S}{4}$.

Стр.219, Зад.15. ΔABC - правоъгълен.

От формулата за лице на триъгълник $S = \left(p - c\right).r_c$ следва, че $S = \left(\dfrac{a+b+c}{2} - c\right).r_c = \left(\dfrac{a+b-c}{2}\right).r_c$, но в правоъгълен триъгълник $\dfrac{a+b-c}{2} = r$, следователно $S = \left(\dfrac{a+b-c}{2}\right).r_c = r.r_c$.